Final Report for Grant No. N00014-94-1-0340
January 1, 1994 – December 31, 1996

Potential Reduction Interior Point Methods for Mixed Nonlinear Complementarity Problems, Nonlinear Programs, and Extensions

by
Renato D.C. Monteiro

School of Industrial and Systems Engineering
Georgia Institute of Technology, Atlanta, GA 30332

I. Summary of Completed Project

A total of fourteen (14) papers and/or reports acknowledge the grant N00014-94-1-0340 among which nine (9) have already been accepted for publication. They are listed at the end of this report.

It should be mentioned that seven (7) other papers acknowledge another related ONR grant, namely N00014-93-1-0234. This grant started in January 1, 1993 while I was an assistant professor at the University of Arizona. Part of the remaining funds for the first year of this grant were transferred to Georgia Tech in the form of a subcontract and remained effective until September 1995. A final report on this grant was submitted during the last quarter of 1995. I will not discuss the contents of these seven papers in this final report since they do not acknowledge the grant N00014-94-1-0340.

II. Main Research

Until December 31 of 1995, which was the official termination date for the grant, the main goal of this project was to study interior-point algorithms for solving the nonlinear complementarity problem (NCP) and its extensions such as the mixed NCP and the constrained nonlinear equation problem, and to obtain more specialized convergence results for the particular case of the mixed NCP which arises from the Karush-Kuhn-Tucker (KKT) conditions of general nonlinear programming problems.

The grant was then renewed for a period of one more year. During this period, the main goal of the project was to study interior-point algorithms for solving linear and nonlinear semidefinite programs and the related (mixed) complementarity problem defined on the cone of positive semidefinite matrices.

Georgia Tech

CONTRACTING SUPPORT DIVISION
404•894•6944

Georgia Institute of Technology
Office of Contract Administration
Atlanta, Georgia 30332-0420 USA

Fax: 404•894•5285

March 21, 1997

Reply to: E-24-X40

Office of Naval Research
Program Officer
Mr. Donald Wagner, Code 311
Office of Naval Research
Ballston Tower One
800 North Quincy Street
Arlington, VA 22217-5660

Subject: Final Technical Report
Project Director(s): R. Monteiro
Telephone No.: (404)894-1450
Contract No.:**N00014-1-0340**
Prime No.:
"POTENTIAL REDUCTION INTERIOR POINT METHODS FOR MIXED NONLINEAR..."
Period Covered: 940101-961231

The subject report is forwarded in conformance with the contract/grant specifications.

Should you have any questions or comments regarding this report(s), please contact the Project Director of the undersigned at 404-894-4764.

/tw

Sincerely,

Wanda W. Simon /tw

Wanda W. Simon
Reports Coordinator

Distribution:
Addressee: 3 copies
1 copy to: Director Naval Research
2 copies to: Defense Technical Info. Center
cc:ONR-RR

The papers [11, 12, 13] are related with the first goal mentioned above while the papers [1, 2, 3, 4, 6, 7, 9, 10] are related with the second goal. I next give more specific comments on each of these works starting with the first set of papers followed by the second one.

In [13], Pang, Wang and I develop a potential reduction interior-point algorithm for the constrained nonlinear equation problem, which provides an excellent framework for understanding primal-dual interior-point algorithms for solving NCPs and nonlinear programs. We then specialize the algorithm and its convergence results to the context of the mixed NCP and the KKT condition associated with a nonlinear program. We have tested the algorithm on various convex programming problems and found that the algorithm is both robust and efficient. This paper appeared in *Mathematical Programming.*

The work [11] studies existence conditions and convergence behavior of the central path for convex programming problems in which both the objective and constraint functions are analytic. This paper will appear in *Computational Optimization and its Applications.*

The paper [12] establishes the superlinear convergence of primal-dual (feasible and infeasible) path following algorithms for solving linearly constrained convex programs satisfying the following conditions: 1) an optimal solution satisfying strict complementarity exists; 2) the Hessian of the objective function satisfies a certain invariance property. This paper will appear in *Computational Optimization and its Applications.*

The paper [1] deals with primal-dual interior-point algorithms for semidefinite programming (SDP) based on two directions which have been introduced by Kojima, Shindoh and Hara. By characterizing these directions as the unique solution of a system of linear equations in symmetric matrix variables, I present a novel approach for obtaining polynomial convergence of several primal-dual path following algorithms for SDP. In particular, we extend for the first time the long-step primal-dual path following linear programming algorithm of Kojima, Mizuno and Yoshise to the context of SDP. (Kojima, Shindoh and Hara mention in section 9 of their paper that they were unable to provide this extension.) This paper will appear in *SIAM Journal on Optimization.*

Paper [10] generalizes the analysis of paper [1] and establishes the polynomial convergence of the long-step path following method based on a whole family of search directions, called Monteiro-Zhang family, which includes the Nesterov-Todd direction and a Newton direction introduced by Kojima, Shindoh and Hara, Helmberg, Rendl,Vanderbei and Wolkowicz, and rediscovered through a different formulation

by myself in the paper [1]. An important consequence of the analysis of this paper is that the long-step version of the Nesterov-Todd path following method has $\mathcal{O}(nL)$ iteration-complexity bound. This paper has been accepted for publication in *Mathematical Programming.*

Papers [2, 9] establish, among other things, the polynomial convergence of algorithms for semidefinite programming based on the Alizadeh, Haeberly and Overton direction. This question has been open for at least 2 years since this direction has been proposed. Many researchers have been trying to solve this problem. I should remark that Prof. Mike Overton, the editor-in-chief of *SIAM Journal on Optimization* (SIOPT), was quite impressed with the paper [2] and invited me to submit it to SIOPT. The other paper [9] has been accepted for publication in *Mathematical Programming.*

In their original paper, Kojima, Shindoh and Hara established the polynomial convergence of the short-step path following algorithm based on only one direction of their family and left as an open question the polynomiality of this algorithm with respect to the other directions of their family. This question has been successfully answered in my paper [7], which establishes an $\mathcal{O}(\sqrt{n}L)$ iteration-complexity bound for the short-step path following algorithm based on any direction of the Kojima, Shindoh and Hara family.

In the paper [6], Tsuchiya and I develop a new class of (feasible) primal-dual interior-point path following algorithms for semidefinite programming (SDP) whose search directions are obtained by applying Newton method to the symmetric central path equation

$$(P^T XP)^{1/2}(P^{-1}SP^{-T})(P^T XP)^{1/2} - \mu I = 0,$$

where P is a nonsingular matrix. Specifically, we show that the short-step path following algorithm based on the Frobenius norm neighborhood and the semilong-step path following algorithm based on the operator 2-norm neighborhood have $O(\sqrt{n}L)$ and $O(nL)$ iteration-complexity bounds, respectively. When $P = I$, this yields the first polynomially convergent semilong-step algorithm based on a pure Newton direction. Restricting the scaling matrix P at each iteration to a certain subset of nonsingular matrices, we are able to establish an $O(n^{3/2}L)$ iteration-complexity for the long-step path following method. Preliminary computational experiment demonstrates that new direction is quite efficient when used in conjunction with primal-dual feasible methods for SDP.

In the papers [3, 4], Pang and I develop interior-point algorithms for solving nonlinear semidefinite programs and the related (mixed) complementarity problem defined

on the cone of positive semidefinite matrices. These two papers generalizes the analysis of two of our previous papers to the context of semidefinite programming. Inspired on our previous paper [13], the paper [4] develops a more general potential reduction interior-point algorithm for solving the constrained nonlinear equation problem, which provides an excellent framework for understanding primal-dual interior-point algorithms for solving nonlinear semidefinite programs and nonlinear complementarity problems in the cone of symmetric positive semi-definite matrices. We then specialize the algorithm and its convergence results to the context of the mixed NCP and the KKT condition associated with a nonlinear semidefinite program. Paper [3] plays a fundamental role on the the analysis of the paper [4]. This paper studies properties of some fundamental interior-point mappings associated with a mixed implicit complementarity problem in the cone of symmetric positive semi-definite matrices. As a consequence of the results of [3], we establish for the first time the existence of the weighted central paths for semidefinite programming problems.

III. Related Research.

I now give a summary of the papers which are not directly related to the current project. There are three of them, namely [5, 8, 14].

I have been involved in a joint research with Dr. Takashi Tsuchiya from the Institute of Statistical Mathematics in Japan for a couple of years. I have co-authored with him a number of papers on affine scaling algorithms for linear and convex quadratic programs. One of these, namely paper [5], acknowledge the current grant. It establishes the global convergence of the second-order affine scaling algorithm for convex quadratic programming. Although the analysis of this paper is an extension of the corresponding one for linear programming given by Tsuchiya and Muramatsu, it is much harder than the later since many of the nice properties enjoyed by LP do not carry over to convex QP. This paper contains many interesting results, including a clear exposition of a matrix decomposition result which has implication to the analysis of other interior point algorithms. This paper will appear in *SIAM Journal on Optimization.*

Jointly with my student Yanhui Wang, I have written two papers [8, 14]. In the work [8], we introduce a trust region affine scaling method for linearly constrained problems and show its convergence for linearly constrained problems which has convex or concave objective functions and whose feasible region is nondegenerate. The paper [14] studies several types of degeneracy for linear programs. The study of nondegeneracy conditions is crucial in the convergence analysis of several algorithms for

linear programs. The papers [8] and [14] will appear in the journals *Mathematical Programming* and *Computational Optimization and its Applications*, respectively.

IV. Miscellaneous

This grant has partially supported four Ph.D. students over different periods of time, namely: Fangjun Zhou, Hamish Warterer, Paulo Zanjacomo and Yanhui Wang. Yanhui Wang graduated in May of 1996 and Fangjun Zhou will be graduating soon. Paulo Zanjacomo has been under my advisory since January of 1996. Hamish Warterer worked with me only temporarily and decided to pursue a different area of research.

I have collaborated with several investigators during the duration of this grant including: Yin Zhang (Rice University), Takashi Tsuchiya (Institute of Statistical Mathematics – Japan) and Jong-Shi Pang (John Hopkins University).

I have visited the Institute of Statistical Mathematics in Japan during the summer of 1996.

I was elected vice-chair of the area of linear programming and complementarity for the new Optimization section of INFORMS. I was also elected cluster chair for the ORSA/TIMS meeting of Phoenix (November 1993) and for the INFORMS meeting of Atlanta (November 1996).

I have been appointed Associate Editor for Operations Research in the beginning of 1996.

References

[1] R. D. C. Monteiro. Primal-dual path following algorithms for semidefinite programming. manuscript, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, September 1995. To appear in *SIAM J. on Optimization.*

[2] R. D. C. Monteiro. Polynomial convergence of primal-dual algorithms for semidefinite programming based on Monteiro and Zhang family of directions. manuscript, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, July 1996.

[3] R. D. C. Monteiro and J.-S. Pang. On two interior-point mappings for nonlinear semidefinite complementarity problems. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, May 1996. Submitted to *Mathematics of Operations Research.*

[4] R. D. C. Monteiro and J.-S. Pang. A newton barrier method for constrained equations. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, March 1997.

[5] R. D. C. Monteiro and T. Tsuchiya. Global convergence of the affine scaling algorithm for convex quadratic programming. Technical Report, School of Industrial Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA, March 1995. To appear in *SIAM Journal on Optimization.*

[6] R. D. C. Monteiro and T. Tsuchiya. Polynomial convergence of a new family of primal-dual algorithms for semidefinite programming. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, November 1996. Submitted to *SIAM Journal on Optimization.*

[7] R. D. C. Monteiro and T. Tsuchiya. Polynomiality of primal-dual algorithms for semidefinite linear complementarity problems based on the Kojima-Shindoh-Hara family of directions. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, August 1996. Submitted to *Mathematical Programming.*

[8] R. D. C. Monteiro and Y. Wang. Trust region affine scaling algorithms for linearly constrained convex and concave programs. Technical Report, School of

ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, June 1995. To appear in *Mathematical Programming.*

[9] R. D. C. Monteiro and P. Zanjacomo. A note on the existence of the Alizadeh-Haeberly-Overton direction for semidefinite programming. manuscript, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, June 1996. To appear in *Mathematical Programming.*

[10] R. D. C. Monteiro and Y. Zhang. A unified analysis for a class of path-following primal-dual interior-point algorithms for semidefinite programming. manuscript, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, June 1996. To appear in *Mathematical Programming.*

[11] R. D. C. Monteiro and F. Zhou. On the existence and convergence of the central path for convex programming and some duality results. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, April 1994. To appear in *Computational Optimization and Applications.*

[12] R. D. C. Monteiro and F. Zhou. On superlinear convergence of infeasible-interior-point algorithms for linearly constrained convex programs. manuscript, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, July 1995. To appear in *Computational Optimization and Applications.*

[13] T. Wang, R. D. C. Monteiro, and J.-S. Pang. An interior point potential reduction method for constrained equations. *Mathematical Programming*, 74:159–195, 1996.

[14] Y. Wang and R. D. C. Monteiro. Nondegeneracy of polyhedra and linear programs. Technical Report, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, August 1994. To appear in *Computational Optimization and Applications.*